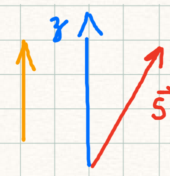
SPIN 1/2 EM CAMPO MAGNÉTICO EXTERNO UNIFORME



$$i\hbar \frac{d|\psi(t)\rangle}{dt} = H|\psi(t)\rangle \quad \text{(EQ. DE SCHRÖEDINGER)} \quad \vec{B}=B_0\hat{z}$$

$$H = -\vec{\mu}\cdot\vec{B}\,;\quad \vec{\mu} = -\frac{ge}{m}\vec{S}\,;\quad \vec{B}=B_0\hat{z}\,;\quad \vec{S}\cdot\vec{B} = S_z B_z\,;$$

$$H = +\frac{ge}{m}\vec{S}\cdot\vec{B} = \frac{ge}{m}S_z B_z = \frac{geB_0}{m}\frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\,; \qquad \text{DEFINIR: } \omega \equiv \frac{geB_0}{2m}$$

O HAMILTONIANO SE ESCREVE:

$$\boxed{H = \frac{\omega\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}}$$

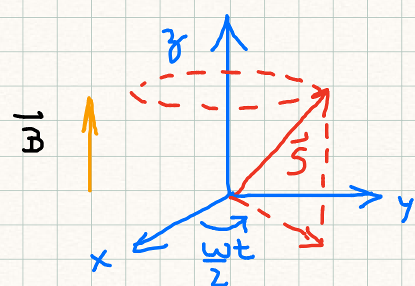
$$|\psi(t)\rangle = a(t)|+\rangle + b(t)|-\rangle = \begin{pmatrix}a(t)\\ b(t)\end{pmatrix}$$

$$i\hbar\frac{d}{dt}\begin{pmatrix}a(t)\\ b(t)\end{pmatrix} = \frac{\omega\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}a(t)\\ b(t)\end{pmatrix}$$

$\dot{} \equiv \frac{d}{dt}$;

$$\dot{a}(t) = -\frac{i\omega}{2}a(t)$$

$$\dot{b}(t) = +\frac{i\omega}{2}b(t)$$



$$a(t) = a(0)\,e^{-i\omega t/2}$$

$$b(t) = b(0)\,e^{i\omega t/2}$$

$$|\psi(t)\rangle = a(0)e^{-i\omega t/2}|+\rangle + b(0)e^{i\omega t/2}|-\rangle\,.$$

OBSERVE QUE:

$$\langle\psi(t)|\psi(t)\rangle = |a(0)|^2 + |b(0)|^2 = \langle\psi(0)|\psi(0)\rangle$$

AS PROBABILIDADES NÃO EVOLUEM NO TEMPO, O SISTEMA É ESTACIONÁRIO.

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SUPOR QUE:

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \ ; \qquad a(0) = b(0) = 1/\sqrt{2}$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} e^{-i\omega t/2} |+\rangle + \frac{1}{\sqrt{2}} e^{i\omega t/2} |-\rangle$$

$$\langle S_z\rangle_t = \langle\psi(t)|S_z|\psi(t)\rangle = \frac{1}{2}\left(\frac{\hbar}{2}\right) + \frac{1}{2}\left(-\frac{\hbar}{2}\right) = 0.$$

$$\langle S_x\rangle_t = \langle\psi(t)|S_x|\psi(t)\rangle$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\omega t/2} \\ e^{i\omega t/2}\end{pmatrix} ; \qquad S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix};$$

$$\langle S_x\rangle_t = \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\cdot\frac{\hbar}{2}\begin{pmatrix} e^{i\omega t/2} & e^{-i\omega t/2}\end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix}\begin{pmatrix} e^{-i\omega t/2} \\ e^{i\omega t/2}\end{pmatrix}$$

$$= \frac{\hbar}{4}\begin{pmatrix} e^{i\omega t/2} & e^{-i\omega t/2}\end{pmatrix}\begin{pmatrix} e^{i\omega t/2} \\ e^{-i\omega t/2}\end{pmatrix}$$

$$= \frac{\hbar}{4}\left(e^{i\omega t} + e^{-i\omega t}\right)$$

$$= \frac{\hbar}{2}\left(\frac{e^{i\omega t} + e^{-i\omega t}}{2}\right)$$

$$= \frac{\hbar}{2}\cos\omega t$$

PROBALIDADES:

$$|{}_x\langle +|\psi(t)\rangle|^2 = ? \quad \text{E} \quad |{}_x\langle -|\psi(t)\rangle|^2 \ ?$$

$$|+\rangle_x = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1\end{pmatrix}$$

$${}_x\langle +|\psi(t)\rangle = \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1\end{pmatrix}\begin{pmatrix} e^{-i\omega t/2} \\ e^{i\omega t/2}\end{pmatrix} = \frac{1}{2}\left(e^{-i\omega t/2} + e^{i\omega t/2}\right)$$

$$= \cos \frac{\omega t}{2} \; ;$$

$$\boxed{\left| {}_x\langle + | \psi(t) \rangle \right|^2 = \cos^2\left(\frac{\omega t}{2}\right)}$$

$$|-\rangle_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

IMPORTANTE P/ NMR, MRI:

P/ TEMPERATURA ZERO K, OS DOIS ESTADOS DE ENERGIA SÃO IGUALMENTE PROVÁVEIS; MAS P/ $T \neq 0$, O ESTADO + BAIXO É LIGEIRAMENTE FAVORECIDO, DE ACORDO COM A RELAÇÃO:

$$\frac{N_1}{N_0} = \exp\left(-\frac{\Delta E}{k_B T}\right)$$

$${}_x\langle - | \psi(t) \rangle = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} (1 \; -1) \begin{pmatrix} e^{-i\omega t/2} \\ e^{i\omega t/2} \end{pmatrix}$$

$$= \frac{1}{2} \left(e^{-i\omega t/2} - e^{i\omega t/2} \right)$$

$$= -i \left(\frac{e^{i\omega t/2} - e^{-i\omega t/2}}{2} \right)$$

$$= -i \, \text{Sen}\left(\frac{\omega t}{2}\right)$$

$$\boxed{\left| {}_x\langle - | \psi(t) \rangle \right|^2 = \text{Sen}^2\left(\frac{\omega t}{2}\right)}$$

$$\left| {}_x\langle - | \psi(t) \rangle \right|^2 + \left| {}_x\langle + | \psi(t) \rangle \right|^2 = \text{Sen}^2\left(\frac{\omega t}{2}\right) + \cos^2\left(\frac{\omega t}{2}\right) = 1 \quad {}_{m}$$

COMO H É DIAGONAL NA REPRESENTAÇÃO $|+\rangle, |-\rangle$, OS AUTO-VALORES DE ENERGIA SÃO:

$$E_1 = \frac{\hbar\omega}{2} \; ; \; E_0 = -\frac{\hbar\omega}{2}$$

SOLUÇÃO VIA O OPERADOR DE EVOLUÇÃO

COMO H NÃO DEPENDE DO TEMPO:

$$|\psi;t\rangle = U_t\,|\psi;t=0\rangle$$

ONDE:

$$U_t = \exp\left(\frac{-iHt}{\hbar}\right)$$

$$|\psi;t=0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \;; \qquad H = \omega S_z$$

$$|\psi;t\rangle = \exp\left(\frac{-iHt}{\hbar}\right)\frac{1}{\sqrt{2}}\big(|+\rangle + |-\rangle\big)$$

$$H|+\rangle = \frac{\omega\hbar}{2}|+\rangle \;; \qquad H|-\rangle = -\frac{\hbar\omega}{2}|-\rangle$$

$$\therefore\quad |\psi;t\rangle = \frac{1}{\sqrt{2}}\left(\exp\left(-\frac{iHt}{\hbar}\right)|+\rangle + \exp\left(-\frac{iHt}{\hbar}\right)|-\rangle\right)$$

$$= \frac{1}{\sqrt{2}}e^{-i\frac{\omega t}{2}}|+\rangle + \frac{1}{\sqrt{2}}e^{-i\frac{\omega}{2}t}|-\rangle$$

QUANDO H NÃO DEPENDE DO TEMPO PODEMOS RESOLVER O PROBLEMA DA EVOLUÇÃO TEMPORAL DO KET DE ESTADO DE 2 MODOS: C/ U_t OU C/ A EQ. DE S. SE H DEPENDE DO TEMPO, U_t TEM UMA FORMA MAIS COMPLEXA, MAS A EQ. DE S. SE ESCREVE:

$$\boxed{i\hbar\frac{d}{dt}|\psi;t\rangle = H(t)\,|\psi;t\rangle}$$

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